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Lowered Beachhead Cost in a Simplified Melitz Model with Heterogeneous Fixed Cost

Hiroyuki Mizuta

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Kushiro Public University

Ashino 4-1-1, Kushiro, Hokkaido, 085-8585, Japan

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Hiroyuki Mizuta[1]

1. Introduction

In the literature of monopolistic competition trade model of heterogeneous firms, there are some features which are different from traditional monopolistic competition trade model.[2]

In the traditional monopolistic competition trade model, all produced goods are consumed, so there are non-traded goods. On the other hand, in heterogeneous firms' literature, fixed transport cost or beachhead cost is incorporated into models.[3] So that, only some firms export their goods, and people cannot consume non-traded goods produced abroad.

Secondly, in heterogeneous firms' literature, lowered iceberg-type transport cost urges firms to export their goods, and in turn, from the increase

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[2]For traditional models, See Krugman (1980), Helpman and Krugman (1985) and Krugman (1991), and for heterogeneous firms' literature, Melitz (2003), Baldwin (2005), Baldwin and Forslid (2006), Demidova (2008), Redding (2010), and Melitz and Trefler (2012).)

[3] Medin (2003) incorporated beachhead cost into a traditional monopolistic competition trade model, and showed that when beachhead cost is high, many firms do not export. But, In his model, no heterogeneity is considered. So, it cannot tell which firms stop export.

of intensity of competitiveness, inefficient firms are sorted out from the domestic production. And, this selection is said to cause gains from trade, in addition to gains from trade based on "love of variety" which is the single source of trade gains in traditional models.

In this heterogeneous firms' literature, this paper will contribute two ideas. First is to construct a Simplified Melitz model, where fixed costs not marginal costs like in Melitz, are assumed to be heterogeneous and beachhead costs are assumed to be proportional to fixed costs. [4] Second is to demonstrate how a specialisation pattern changes sequentially as beachhead cost changes. As beachhead cost can be considered as a proxy of iceberg type transport cost, this will show a connection between traditional models and heterogeneous firms' literature.[5]

A motive for creating a model with heterogeneous fixed cost and beachhead cost proportional to it is to simplify the analysis of lowered iceberg type trade cost. Lowering iceberg-type transport cost lowers prices of exported goods abroad, which increases export and the profit from export, which is indirect. On the other hand, lowering beachhead cost directly increases the profit from export.[6] Thus, beachhead cost is regarded as a proxy of the iceberg transport cost, and, lowering beachhead cost makes analysis easier than iceberg

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[4] Kikuchi (1996) wrote one of models which include different fixed cost in traditional models. His model is different in the way of introducing heterogeneity, from heterogeneous firms' literature. Different goods have to have different index a in his model, while in heterogeneous firms' literature even different goods such as an orange and a light bulb are members of the same a group if their marginal costs take the same value.

[5] As far as I know, Falvey, Greenaway and Yu (2006) is the first paper which considered various specialisation patterns in Melitz model.

[6] For treatment of profit in monopolistic competition trade model, see Picard, Thisse and Toulemonde (2002).

-type transport cost, so, it is helpful even in considering the case of lowering iceberg transport cost.

This paper is organised as follows. In sec. 2, assumptions of the model are explained. In the next sec. 3, patterns of specialisation are classified according to the comparison of two values a_D and $\tilde{a}_D$ or $\tilde{a}_X$ and $\tilde{a}_D$, and a value of an index of beachhead cost, k, for each patterns is calculated. In sec. 4, concluding remarks, I compare the case of this paper that in small country, no firms emerge when beachhead cost is valued in the middle range with the similar results that in a traditional trade model, which are obtained in Mizuta (2013).

2. Model

Two types of goods, manufactured goods and only a single agricultural product, are traded between larger home and smaller foreign countries, with home scale L larger than that of foreign, $L > \tilde{L}$. For the agricultural product, any trading costs are not burdened, while for manufactured goods or simply goods, its producers have to pay fixed export cost or beachhead cost, when they export their goods.

People in countries have the utility of $U = (\int_0^b c(z)^\rho dz)^s Y^{1-s}$ from their consumption of each goods by the amount of $c(z)$, and of agricultural product by the amount of Y, so that shares of income spent on goods and agricultural product are s and $1-s$, respectively.

Each manufacturer, or firm, draws an index of fixed cost a in a lottery in advance of their entry into market, or start of operation. Its outcomes range from 0 to $+\infty$. Firms that had drawn a large value of the index of fixed cost a are finding its operation start unprofitable, so that actually no more than firms with 0 to a_D for home country, and 0 to $\tilde{a}_D$ for foreign one, engage in producing goods. The number of firms which actually operate in each country are written n and $\tilde{n}$, for home and foreign country respectively. Upon export, each firms faces beachhead cost assumed proportional to its fixed cost, that is, to be ka for firms with its fixed cost a , so that further smaller parts of the firms which engage in production notice that export is profitable, and begin exporting. The density of a and the cumulative distribution at a are written $g(a)$ and $G(a)$.

One unit of agricultural product is defined as a numeraire, with each unit priced at 1. For each firm producing goods, its marginal cost is 1, and its burden from production comprises from fixed and variable costs. Firms are under monopolistic competition, with all firms pricing their goods at $p=1/\rho$.

In the subsequent sections which follow, I will explain the relation between specialisation patterns, in the sense of whether all firms engage in export or not, and beachhead cost. And the latter could be considered to be lowered through international agreements or improvements of transporting technologies.

3. Specialisation

On considering whether to export or not to export, each firms has to take beachhead costs, such as building a distribution centre overseas, into consideration. It is a hurdle for firms needing to operate abroad. Those firms which overcome the hurdle are the only ones able to engage in export. Melitz (2003) introduced beachhead costs to demonstrate the phenomenon that only some firms engage in trade, and since then, only this case has been dealt with in heterogeneous firms' literature. But, as shown in this paper, when beachhead cost is low, traditional assumption that all firms engage in export instead appears again.

Furthermore, if the home country is larger the foreign country, the foreign country is more attractive to firms from the point of export. So, the first country where all firms start to export as beachhead cost decreases seem to be foreign country. But, the story is not so simple. firms go out of business in this case from the foreign country. In the following sections, I explain these three cases in order.

3.a. The case of $a_X < a_D$ and $\tilde{a}_X < \tilde{a}_D$

In this subsection, I will explain the case that in both countries only some firms engage in production, export, $a_X < a_D$ and $\tilde{a}_X < \tilde{a}_D$. Net profit from export, of each home firm, is written as $(1-\rho)s\tilde{L}/(nG(a_X)/G(a_D)+\tilde{n})-ka$, where $nG(a_X)/G(a_D)$ is the number of home firms exporting their goods to a foreign country. So, when k is high, some firms with higher fixed cost cannot obtain any profits from export, so that they specialise only in selling in its domestic market. The same applies to foreign firms. So, when k is high, the case similar to that treated by Melitz (2003) emerges.

Zero profit conditions of home and foreign firms in two countries' markets are written as

$$\frac{(1-\rho)s\tilde{L}}{nG(a_X)/G(a_D)+\tilde{n}} = ka_X = \tilde{a}_D\ , \qquad \frac{(1-\rho)sL}{n+G(\tilde{a}_X)/G(\tilde{a}_D)\tilde{n}} = a_D = k\tilde{a}_X\ . \tag{1}$$

From Eqs.(1) and $a_X < a_D$ and $\tilde{a}_X < \tilde{a}_D$, the following inequalities follow.

$$\tilde{a}_D / k = a_X < a_D = k\tilde{a}_X < k\tilde{a}_D\ ,\ k > 1\ . \tag{2}$$

In this section, I am only dealing with k larger than 1.

Firms draw an index of fixed cost a in a lottery. So that, if any firms enters into the market, free entry conditions have to be satisfied for all firms in both countries,

$$\begin{aligned} f_E &= \int_0^{a_D} \pi(a)da + \int_0^{a_X} \pi_X(a)g(a)da \\ &= \int_0^{a_D} G(a)da + k\int_0^{a_X} G(a)da \\ &= Q(a_X, a_D; k) \end{aligned}\ , \text{(3a)} \qquad \begin{aligned} f_E &= \int_0^{\tilde{a}_D} G(\tilde{a})d\tilde{a} + k\int_0^{\tilde{a}_X} G(\tilde{a})d\tilde{a} \\ &= Q(\tilde{a}_X, \tilde{a}_D; k) \end{aligned}\ . \text{(3b)}$$

And, if free entry conditions become unbinding it is expected that profit will become smaller than the entry cost, if no firms in either country would enter into the market.

In this section, I explain cases i) $n > 0$ and $\tilde{n} > 0$ and ii) $n > 0$ and $\tilde{n} = 0$ and thirdly that iii) there is no case of $n = 0$ and $\tilde{n} > 0$.

3.a.i. $n > 0$ and $\tilde{n} > 0$

Using Eq.(1), as Eq.(3) are symmetric in a_X and $\tilde{a}_X$ or a_D and $\tilde{a}_D$,

$$a_D = \tilde{a}_D = ka_X = k\tilde{a}_X \qquad (4)$$

holds. And, Eqs. (3a) and (3b), are written to be $Q(a_X, ka_X; k) = f_E$, with a_X and a_D decreasing and increasing, respectively, as k increases:

$$\frac{da_X}{dk} = -\frac{a_X G(ka_X) + \int_0^{a_X} G(a)da}{kG(ka_X) + G(a_X)} < 0 \quad \text{(5a)}, \quad \frac{da_D}{dk} = \frac{d(ka_X)}{dk} = \frac{\int_0^{a_X} adG}{G(ka_X) + G(a_X)} > 0 \text{ .(5b)}$$

This implies the next proposition for sorting out which is similar to that in heterogeneous firms' literature.

Proposition 1

An increase of beachhead cost sorts inefficient firms with high fixed cost, out of export, and less competition from it allows less efficient firms to remain in production.

On the other hand, from Eq.(1), for home and foreign numbers of firms to be positive,

$$\frac{k\tilde{a}_X G(\tilde{a}_X)}{\tilde{a}_D G(\tilde{a}_D)} < \frac{L}{\tilde{L}}, \qquad \frac{ka_X G(a_X)}{a_D G(a_D)} < \frac{\tilde{L}}{L} \qquad (6)$$

have to hold. This is expressed as in the next proposition 2,

Proposition 2

The ratio of total export to domestic sales for both countries, must be less than the ratio of the scale of the destination country for export to that of a domestic market.

Proposition 2 means that small export has to make some room for local firms in a destination country for export, to operate.

From Eqs. (2),(4), (6), and $L/\tilde{L}>1$, in Subsubsec. 3.a.i, the next relation has to hold.

$$k>1, G(ka_X)/G(a_X)>L/\tilde{L}\ . \tag{7}$$

Furthermore, $G(ka_X)/G(a_X)$ increases with k as

$$\frac{d}{dk}\frac{G(ka_X)}{G(a_X)}=\frac{kg(ka_X)G(a_X)\int_0^{a_X} adG(a)+G(ka_X)g(a_X)(\int_0^{a_X} G(a)da+a_XG(ka_X))}{kG(a_X)^2G(ka_X)+G(a_X)}>0\ . \tag{8}$$

And, as $\lim_{k\to+\infty} G(a_D)/G(a_X)=+\infty$ and $\lim_{k\to+0} G(a_D)/G(a_X)=0$ hold, Lemma 1 holds.

Lemma 1

There is $k^*>1$ such that $G(k^*a_X)/G(a_X)=L/\tilde{L}$ hold at a_X and a_D satisfying Eqs. (3) and (4). And, For any $k\in(k^*,\infty)$, Ineqs.(7) hold. And, for any $k<k^*$, $G(ka_X)/G(a_X)<L/\tilde{L}$ holds.[7]

[7] For proofs of $\lim_{k\to+\infty} G(a_D)/G(a_X)=+\infty$ and $\lim_{k\to+0} G(a_D)/G(a_X)=0$, see Appendix 3-1.

Furthermore, as the ratio of the number of firms in home country to that in foreign country is written as $n/\tilde{n} = \left(L - \tilde{L}G(a_X)/G(a_D)\right) \big/ \left(\tilde{L} - LG(a_X)/G(a_D)\right)$,

$$\frac{d(n/\tilde{n})}{d(G(a_X)/G(a_D))} \cdot \frac{G(a_X)/G(a_D)}{n/\tilde{n}} = \frac{L^2 - \tilde{L}^2}{(L - \tilde{L}G(a_X)/G(a_D))((\tilde{L} - LG(a_X)/G(a_D))} > 0,$$

holds for $k < k^*$. Using Eq. (8), this equation implies the following proposition.

Proposition 3

Beachhead cost k decreases, the ratio of the number of firms in larger home country to smaller foreign $n/\tilde{n}$ increases.

This relation of beachhead cost and the ratio of the number of firms corresponds to the home market effect in Helpman and Krugman (1985, pp. 205-209). In their model, a decrease of iceberg transport cost increases the ratio between the number of firms in large country to that in small country. [8] Through this 'home market effect,' the number of firms in small country decreases to zero as k decreases to k^*.

3.a.ii. $n > 0$ and $\tilde{n} = 0$

When $\tilde{n} = 0$, next equations follow from Eqs.(1),

[8] The original home market effect is the relation between the ratio of the scale of two countries and the ratio of the numbers of firms in large and small countries. There, a decrease of iceberg type transport cost increases the ratio of the number of firms in a large country to that in a small country.

$$\frac{(1-\rho)s\tilde{L}}{nG(a_X)/G(a_D)} = ka_X = \tilde{a}_D \quad , \frac{(1-\rho)sL}{n} = k\tilde{a}_X = a_D \ , \tag{9}$$

$$\frac{G(a_D)}{G(a_X)}\frac{a_D}{a_X} = k\frac{L}{\tilde{L}} \ . \tag{10}$$

On the other hand, using Eqs. (4), Eqs. (3) are modified into

$Q(a_X, a_D, k) = f_E$ (11a), $Q(a_D/k, ka_X, k) = f_E$ (11b).[9]

The slope of Eq.(11a) is smaller than that of Eq.(11b) as

$$-da_D/da_X\big|_{\text{of (11a)}} = kG(a_X)/G(a_D) < kG(ka_X)/G(a_D/k) = -da_D/da_X\big|_{\text{of(11b)}} \ . \tag{12}$$

And, line $a_D = ka_X$ passes through the point of intersection of Eq. (11). At any a_X, there is a relation in the next lemma, between a_D satisfying $a_D = ka_X$ and that satisfying Eq. (10).

Lemma 2

a_D satisfying $a_D = ka_X$ is smaller (larger) than that satisfying Eq. (10), when k is smaller (larger) than k^*

This is shown to hold in the following way: Plugging $a_D = ka_X$ into $G(a_D)a_D/G(a_X)a_X$, $kG(ka_X)/G(a_X)$ is obtained. And, if it is smaller than $kL/\tilde{L}$, a_D

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[9] When $\tilde{n} = 0$ holds, Eqs. (11) are not equations which determine relevant a_X and a_D as Eq.(3b) is unbinding.

which satisfies Eq.(10) is larger than $a_D = ka_X$ at any given a_X and $G(ka_X)/G(a_X) < L/\tilde{L}$ holds. Suppose that a_X is that at the intersection point of Eqs. (11), then this holds when $k < k^*$.

From the above results, Figure 1 is drawn, where foreign firms' expected profit attains the level of their entry cost at the point F, but actually, a_X and a_D are determined at the point E, where foreign expected profit is less than entry cost, so that firms go out of business in a foreign country.

Furthermore, from Lemma 2, next Lemma 3 holds.

Lemma 3

A value of $G(a_D)/G(a_X)$ at a_D and a_X satisfying Eqs. (3) and (4) is smaller (larger) than that at a_D and a_X satisfying Eqs. (3a) and (10), when k is smaller (larger) than k^*.

3.a.iii. $n = 0$ and $\tilde{n} > 0$

In this subsubsenction, I show that $n = 0$ and $\tilde{n} > 0$ does not occur.

Substituting $n = 0$ into Eq. (1), zero profit conditions are written as follows.

$$\frac{(1-\rho)s\tilde{L}}{\tilde{n}} = ka_X = \tilde{a}_D \, , \; \frac{(1-\rho)sL}{G(\tilde{a}_X)/G(\tilde{a}_D)\tilde{n}} = a_D = k\tilde{a}_X \, . \qquad (13)$$

At the intersection point of Eqs. (3), Eqs. (4) holds. But, on the other hand, with Eq. (13), Eqs. (3) are modified into

$$\int_0^{k\tilde{a}_X} G(a)da + k\int_0^{\tilde{a}_D/k} G(a)da = f_E \text{ (14a)}, \int_0^{\tilde{a}_D} G(a)da + k\int_0^{\tilde{a}_X} G(a)da = f_E \text{ .(14b)}$$

As $k > 1$ holds from Eqs. (13), $a_X < a_D$ and $\tilde{a}_X < \tilde{a}_D$, the slope of Eq. (14a) is steeper than that of Eq. (14b), at their intersection point,

$$-d\tilde{a}_D / d\tilde{a}_X\big|_{\text{of (14a)}} = kG(k\tilde{a}_X)/G(\tilde{a}_D/k) > kG(\tilde{a}_X)/G(\tilde{a}_D) = -d\tilde{a}_D / d\tilde{a}_X\big|_{\text{of(14b)}} \qquad (15)$$

On other hand, from Eq.(13), the next equation follows,

$$\frac{L}{\tilde{L}}\frac{G(\tilde{a}_D)}{G(\tilde{a}_X)}\frac{\tilde{a}_D}{\tilde{a}_X} = k \ . \qquad (16)$$

Comparing the value of $\tilde{a}_D$ in $\tilde{a}_D = k\tilde{a}_X$ and that of Eq. (16) at the value of $\tilde{a}_X$ at the intersection of Eqs. (14) , the former is larger than the latter if the next inequality holds.

$$\frac{\tilde{L}}{L} < \frac{G(k\tilde{a}_X)}{G(\tilde{a}_X)} \ . \qquad (17)$$

As $\tilde{L} < L$, Ineq.(17) always holds as $k > 1$. So, $\tilde{a}_D$ of Eq. (16) is smaller than that of $\tilde{a}_D = k\tilde{a}_X$ at the $\tilde{a}_X$ of the intersection point of Eqs. (14). See Figure 2. The intersection point of curves of Eq.(16) and Eq.(14b), Point J, which determines relevant $\tilde{a}_X$ and $\tilde{a}_D$ is in the outside of Point H, which is the intersection point of Eq. (14a) and Eq. (16). This means that expected profit of home firms is larger than entry cost, so that this case is not consistent with the precondition of this subsubsection, $n = 0$. Thus, the case in this subsubsection is not possible.

3.b. The case of $a_D < a_X$ and $\tilde{a}_D < \tilde{a}_X$

In heterogeneous firms' literature, in both countries, there are some firms which are not engage in trade, that is only the case of 3.a is considered. When beachhead cost is low, traditional assumption that all firms engage in export as seen in Krugman (1981), hold. In this subsection, I deal with this case.

This case is when $a_D < a_X$ and $\tilde{a}_D < \tilde{a}_X$. Firms with an index of fixed cost a_X or $\tilde{a}_X$ are those with zero profit from export, but they do not produce at all, so that they cannot engage in export at all, implying a_X and $\tilde{a}_X$ firms are merely imaginary firms. This is simple but sometimes might be difficult to understand; goods not produced cannot be exported.

Each home firms with a obtains its net profit of $(1-\rho)s\tilde{L}/(n+\tilde{n}) - ka$ from its export, and $(1-\rho)sL/(n+\tilde{n}) - a$ from its domestic sales. Zero profit conditions of home and foreign firms are, respectively,

$$\frac{(1-\rho)s\tilde{L}}{n+\tilde{n}} = ka_X = \tilde{a}_D - k(\tilde{a}_X - \tilde{a}_D), \quad \frac{(1-\rho)sL}{n+\tilde{n}} = k\tilde{a}_X = a_D - k(a_X - a_D), \tag{18}$$

where $k(a_X - a_D)$ is net profit from export, for home firms with a_D .

Free entry conditions for firms in home and foreign countries are written respectively to be

$$f_E = (1+k)\int_0^{a_D} G(a)da = Q(a_D, a_D; k) \quad , \quad f_E = (1+k)\int_0^{\tilde{a}_D} G(a)da = Q(\tilde{a}_D, \tilde{a}_D; k) \quad . \tag{19}$$

From Eqs. (18), between a_X and a_D, there is a relation of $a_X = a_D(1+k)/k \cdot \tilde{L}/(L+\tilde{L})$ and a similar relation holds between $\tilde{a}_X$ and $\tilde{a}_D$, and as from Eqs.(19) a_D and $\tilde{a}_D$ take the same value $a_D = \tilde{a}_D$, Thus, a_X and $\tilde{a}_X$ has a relation of $a_X / \tilde{a}_X = \tilde{L}/L < 1$. The next Proposition 4 and Lemma 4 hold.

Proposition 4

When all firms engage in export, even if two firms in large and small countries respectively have the same fixed cost a, firms in a small country gains larger profit from export than those in large country, that is $k(\tilde{a}_X - a) > k(a_X - a)$.

Lemma 4

$k < \tilde{L}/L$ has to be satisfied for the case of $a_D < a_X$ and $\tilde{a}_D < \tilde{a}_X$.[10]

This lemma comes from the fact that $a_D < a_X$ ($\tilde{a}_D < \tilde{a}_X$) holds when $k < \tilde{L}/L$ ($k < L/\tilde{L}$), and $L > \tilde{L}$.

[10] Classification depending on Eqs. (19) are unbinding or not is not necessary as they are the same equations. For analysis in more detail, which include analysis of gains from trade, in the case of $a_D < a_X$ and $\tilde{a}_D < \tilde{a}_X$, see Mizuta (2011)

3.c. The case of $a_X < a_D$ and $\tilde{a}_D < \tilde{a}_X$

In this subsection, the case that beachhead cost k is assumed to be so low that all firms in one country engages in trade, but high enough as some firms in another country refrain from export. Two cases that $a_X < a_D$ and $\tilde{a}_D < \tilde{a}_X$ and that $a_D < a_X$ and $\tilde{a}_X < \tilde{a}_D$ are explained here. I begin this subsection with the example that the latter case does not occur.

Using proof of contradiction, assume that $a_D < a_X$ and $\tilde{a}_X < \tilde{a}_D$ hold. Zero profit conditions are written as

$$\frac{(1-\rho)s\tilde{L}}{n+\tilde{n}} = ka_X = \tilde{a}_D \quad \text{(20a)}, \quad \frac{(1-\rho)sL}{n+\tilde{n}G(\tilde{a}_X)/G(\tilde{a}_D)} = a_D - \left(\frac{(1-\rho)s\tilde{L}}{n+\tilde{n}} - ka_D\right) = k\tilde{a}_X \text{ . (20b)}$$

From Eqs. (20), $a_D < a_X$ and $\tilde{a}_X < \tilde{a}_D$ are calculated respectively as

$$1 > \frac{\tilde{L}}{L} > \frac{\tilde{L}(n+\tilde{n}G(\tilde{a}_X)/G(\tilde{a}_D))}{L(n+\tilde{n})} > k \quad \text{and} \quad 1 < \frac{L}{\tilde{L}} < \frac{L}{\tilde{L}}\frac{n+\tilde{n}}{n+\tilde{n}G(\tilde{a}_X)/G(\tilde{a}_D)} < k \text{ , which}$$

contradict each other. Thus, the case that $a_D < a_X$ and $\tilde{a}_X < \tilde{a}_D$ does not occur.

Proposition 5

When there are non-exporting firms in a small country, it is not possible that all firms in a large country engage in export.

Next, I explain the case of $a_X < a_D$ and $\tilde{a}_D < \tilde{a}_X$, and show that no firm can operate in foreign country: $\tilde{n} = 0$.

Zero profit conditions are

$$\frac{(1-\rho)s\tilde{L}}{nG(a_X)/G(a_D)+\tilde{n}} = ka_X = \tilde{a}_D - k(\tilde{a}_X - \tilde{a}_D) \text{ (21a) }, \frac{(1-\rho)sL}{n+\tilde{n}} = k\tilde{a}_X = a_D \text{ (21b).}$$

From Eqs. (21), $\tilde{a}_D$ is expressed as a weighted average of a_X and a_D,

$$\tilde{a}_D = \frac{1}{1+k}a_D + \frac{k}{1+k}a_X . \qquad (22)$$

Free entry conditions of home firms is written to be

$$f_E = \int_0^{a_D} G(a)da + k\int_0^{a_X} G(a)da = Q(a_X, a_D; k) , \qquad (23)$$

Then, expected profit of foreign firms $(1+k)\int_0^{\tilde{a}_D} G(\tilde{a})d\tilde{a}$ is less than expected profit of the home country $\int_0^{a_D} G(a)da + k\int_0^{a_X} G(a)da$ from Eq.(22) and the fact that $\int_0^{t} G(a)da$ is a strictly convex function of t when density function $g(a)$ is positive at any a, which I assume in this paper. Thus, for foreign firms, expected profit does not attain the entry cost, so that they do not enter the market through drawing an index of fixed cost a in a lottery. Assuming that all firms go out of business at the end of a period, goods are not produced in the foreign country, so that $\tilde{n}=0$.[11]

Plugging $\tilde{n}=0$ into Eqs. (21), next equations are obtained,

$$\frac{(1-\rho)s\tilde{L}}{nG(a_X)/G(a_D)}=ka_X=\tilde{a}_D-k(\tilde{a}_X-\tilde{a}_D)\ ,\ \frac{(1-\rho)sL}{n}=k\tilde{a}_X=a_D\ , \tag{24}$$

$$\frac{G(a_D)}{G(a_X)}\frac{a_D}{a_X}=k\frac{L}{\tilde{L}}\ . \tag{25}$$

Using these equations, for $a_X<a_D$ and $\tilde{a}_D<\tilde{a}_X$ to hold,

$$k>\tilde{L}/L \quad \text{(26a)},\ \frac{G(a_D)}{G(a_X)}<\frac{1}{k}\frac{L}{\tilde{L}} \quad \text{(26b)}$$

have to hold respectively.[12]

I consider the range of k which satisfies Ineq. (26b). Following lemma can be used.

Lemma 5

There is a unique value k^{**} such as $\frac{G(a_D)}{G(a_X)}=\frac{1}{k^{**}}\frac{L}{\tilde{L}}$ holds at a_X and a_D satisfying Eqs. (23) and (25), and k^{**} is in between 1 and k^{*}.

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[11] This assumption was used in Helpman, Melitz, and Yeaple (2004), and Falvey, Greenaway and Yu (2006).

[12] See Appendix 3-2 for the proof of Eq. (26b).

From this lemma, the range of k which satisfies Ineq. (26b) is $k < k^{**}$.

This lemma is proved using Lemma 1, 2 and 3 as follows. First, at $k = k^{*}$, the value of $G(a_D)/G(a_X)$ at a_D and a_X satisfying Eqs.(23) and (25) is equal to $G(a_D)/G(a_X)$ at a_D and a_X satisfying Eqs.(3) and (4) from Lemma 3, so from Lemma 1, the value of $G(a_D)/G(a_X)$ at a_D and a_X satisfying Eqs.(23) and (25) becomes $L/\tilde{L}$. On the other hand, the value of $G(a_D)/G(a_X)$ determined by Eqs. (23) and (25) is an increasing function of k as

$$\frac{d}{dk}\frac{G(a_D)}{G(a_X)} = \frac{g(a_D)G(a_X)^2\left\{Lk\int_0^{a_X} a\,dg + \tilde{L}\int_0^{a_X} G(a)da + LG(a_X)a_X\right\}}{G(a_X)^2} > 0 . \quad (27)$$

As $k^{*} > 1$ holds from Lemma 1, the value $G(a_D)/G(a_X)$ determined by Eqs.(23) and (25) is less than $L/\tilde{L}$ at $k = 1$.

$L/k\tilde{L}$ takes $L/\tilde{L}$ at $k = 1$ and decreases as k increases. Thus, k^{**} such that $G(a_D)/G(a_X) = L/k^{**}\tilde{L}$ holds at a_D and a_X which satisfies Eq. (23) and (25), exists uniquely, between 1 and k^{*}.

And, at the value of $k < k^{**}$, Ineq. (26b) is satisfied.

4. Concluding remarks.

In this paper, I show that the next relation holds between k and patterns of specialisation.

At $k < \tilde{L}/L$, $a_D < a_X$ and $\tilde{a}_D < \tilde{a}_X$ (3.b)

At $k \in (\tilde{L}/L, k^{**})$, $a_X < a_D$, $\tilde{a}_D < \tilde{a}_X$ and $\tilde{n} = 0$. (3.c)

At $k \in (k^{**}, k^{*})$, $a_X < a_D$, $\tilde{a}_X < \tilde{a}_D$ and $\tilde{n} = 0$. (3.a.ii)

At $k \in (k^{*}, +\infty)$, $a_X < a_D$, $\tilde{a}_X < \tilde{a}_D$ and $n > 0$ and $\tilde{n} > 0$. (3.a.i)

At an intermediate value $k \in (\tilde{L}/L, k^{*})$ there are no firms in a small country.

In these concluding remarks, I will compare this results with that of my previous paper (Mizuta 2013) which deals with specialisation in the model with a single homogeneous good but with iceberg-type trade cost.[13] If the sales of firms are less than that which attains zero profit, all firms get out of business even in traditional model. In section 3, Fig. 4 and 5, of my previous paper, cases where a small country produces no manufactured goods are examined.

A result of Fig. 4 is this: When the expenditure share on manufactured goods, s , is smaller than a half, as freeness τ increases, that is iceberg transport cost decreases, changes of pattern of specialisation is from there being firms in both countries to a small country losing all firms.

And, results of Fig. 5 are as follows. When s is larger than a half, and a relative scale of home country γ is larger than the value of s , changes of pattern of specialisation is the same one as I explained above, but when γ is

[13] This is a core model in traditional trade model with which Helpman and Krugman (1985) demonstrated home market effect, in Section10.4 of their book.

between s and γ^* which is less than s , as iceberg transport cost is lowered, firms go out of business at one point, and later, when the transport cost becomes enough small, it begin its operation again. And, when γ is small than γ^* , in both countries, firms remains in operation at any level of transport cost.

The phenomenon that in the middle range of transport cost, firms in a small country go out of business is common between the traditional trade model and firm heterogeneity model, but I think their mechanisms are different.

In traditional trade model, from a lack of expenditure on each goods, profit of all firms in a small country becomes negative simultaneously, and all firms stop their operation. However, when there are firms heterogeneous in cost structure, among heterogeneous firms adjustment on a lack of expenditures on each goods is done gradually. There is not a case that from a lack of expenditures on firms, all firms stop their operation simultaneously. At first, most inefficient firms stop their operation, then relatively inefficient firms stop, while most efficient firms continue their operation. The mechanism through which firms in a small country stop all manufacturing is through free entry condition. Free entry condition is common among all firms in a country, so when a situation is unprofitable to an individual firm, it is unprofitable to all firms, then, then firms go out of business simultaneously.

Appendix

Appendix 3-1. :Proofs of $\lim_{k\to+\infty} G(a_D)/G(a_X)=+\infty$ and $\lim_{k\to+0} G(a_D)/G(a_X)=0$ in Subsubsec. 3.a.i.

a_X which satisfies $\int_0^{ka_X} G(a)da + k\int_0^{ka_X} G(a)da = f_E$, has the next properties

a) As $k\to+\infty$, $a_X\to+0$, b) as $k\to+0$, $a_X\to+\infty$.

After the proof of a) and b), Appendix 3-1 is proved.

Proof of a)

a_X decreases as k increases from Eq. (5a). Suppose that as $k\to+\infty$, $a_X\to\alpha>0$, where α is a constant, which has a positive sign.

Then, $ka_X > k\alpha$ holds, which implies that

$$f_E = \int_0^{ka_X} G(a)da + k\int_0^{a_X} G(a)da > \int_0^{k\alpha} G(a)da + k\int_0^{\alpha} G(a)da.$$

As $k\to+\infty$, the right-hand side of the above equation becomes $+\infty$, which contradicts the above inequality. So, $\alpha\le 0$. However, as $a_X\ge 0$, $\alpha\ge 0$. Thus, $\alpha=0$.

Proof of b)

Suppose that as $k\to+0$, $a_X\to\beta>0$, where β is a constant with a positive sign.

Then, as $k\to+0$, $ka_X\to 0\cdot\beta=0$. From Ineq. (5a), $\beta\ge a_X$, and from (5b) $0\cdot\beta\le ka_X$.

As $k\to+0$, $\int_0^{ka_X} G(a)da + k\int_0^{a_X} G(a)da \to \int_0^{0} G(a)da + 0\cdot\int_0^{\beta} G(a)da = 0$, Then, $0 < f_E = \lim_{k \to +0 > 0} \int_0^{ka_X} G(a)da + \int_0^{a_X} G(a)da$ is not satisfied. So, β has to take $+\infty$.

<u>Proof of Appendix 3-1</u>

From a), for $G(a_D)/G(a_X)$, its denominator takes $+0$, as $k \to +\infty$, and its numerator is non-negative and strictly increases as k increases.

Thus, $\lim_{k \to +\infty} G(a_D)/G(a_X) = +\infty$ follows. $\lim_{k \to +0} G(a_D)/G(a_X) = 0$ is similarly proved.

<u>Appendix 3-2</u>: (26b) has to hold for $a_X < a_D$ and $\tilde{a}_D < \tilde{a}_X$ to hold.

From $k\tilde{a}_X = a_D$ in Eq (21b) and Eq. (22), $0 < \tilde{a}_X - \tilde{a}_D = \dfrac{1}{k(1+k)}(a_D - k^2 a_X)$ holds, which implies that $a_D > k^2 a_X$ is necessary, and from this inequality and Eq. (25) next equation holds,

$$\tilde{L} / L = ka_X G(a_X) / a_D G(a_D) < ka_X G(a_X) / k^2 a_X G(a_D) = G(a_X) / kG(a_D) \ .$$

Thus for $\tilde{a}_D < \tilde{a}_X$ to be satisfied, (26b) has to hold.

Reference

Baldwin E. R. (2005), "Heterogeneous Firms and Trade: Testable and Untestable Properties of the Melitz Model," NBER Working Paper Series, No. w11471.

Baldwin E. R. and R. Forslid, (2006), "Trade Liberalization with Heterogenous Firms," NBER Working Paper, No. w12192.

Demidova S. (2008), "Productivity Improvements and Falling Trade Costs: Boon or Bane?" International Economic Review, vol. 49 (4), pp. 1437-1462.

Falvey R. Greenaway D, and Yu Z, (2006), "Extending the Melitz Model to Asymmetric Countries," University of Nottingham Research Paper Series No. 2006/07.

Helpman E. and P. Krugman (1985), Market Structure and Foreign Trade: Increasing returns, imperfect competition and the international economy, MIT Press, Cambridge, MA.

Helpman E., Melitz J M, and S. Yeaple, (2004), "Export Versus FDI with Heterogeneous Firms" American Economic Review, vol. 94 (1), pp. 300-316.

Kikuchi T. (1996), "Increasing Costs in Product Diversification and Gains from Trade," Japanese Economic Review vol 47 (4), pp.-384-395.

Krugman P. (1980), "Scale Economies, Product differentiation, and the Pattern of Trade," American Economic Review, vol. 70 (5), pp. 950-959.

Krugman P. (1991), "Increasing Returns and Economic Geography," Journal of Political Economy, vol. 99 (3), pp. 483-499.

Medin H, (2003), “Firm’s Export Decisions -Fixed Costs and the Size of the Export Maket,” Journal of International Economics, vol. 61 (1), pp. 225-241.

Melitz M. (2003), “The Impact on Intra-Industry Reallocations and Aggregate Industry,” Econometrica, vol. 71(6), pp. 1695-1725.

Melitz M. and D. Trefler (2012), “Gains from Trade when Firms Matter,” Journal of Economic Perspective, vol. 26 (2), pp. 91-118.

Mizuta H. (2011), “Lowered Beachhead Cost,” Social Sciences , vol. 24, pp. 125-135, (Kushiro Public University).

Mizuta H. (2013), “Specialisation and Centripetal Forces,” Kushiro Public University Discussion Paper, Series A, No. 27.

Picard, P. M., Thisse, J. F, and Toulemonde, E., (2002), "Economic Geography and the Role of Profits," CEPR Discussion Paper, No. 3385.

Redding S. (2010), “Theories of Heterogeneous Firms and Trade,” NBER Working Paper Series, No. w16562.

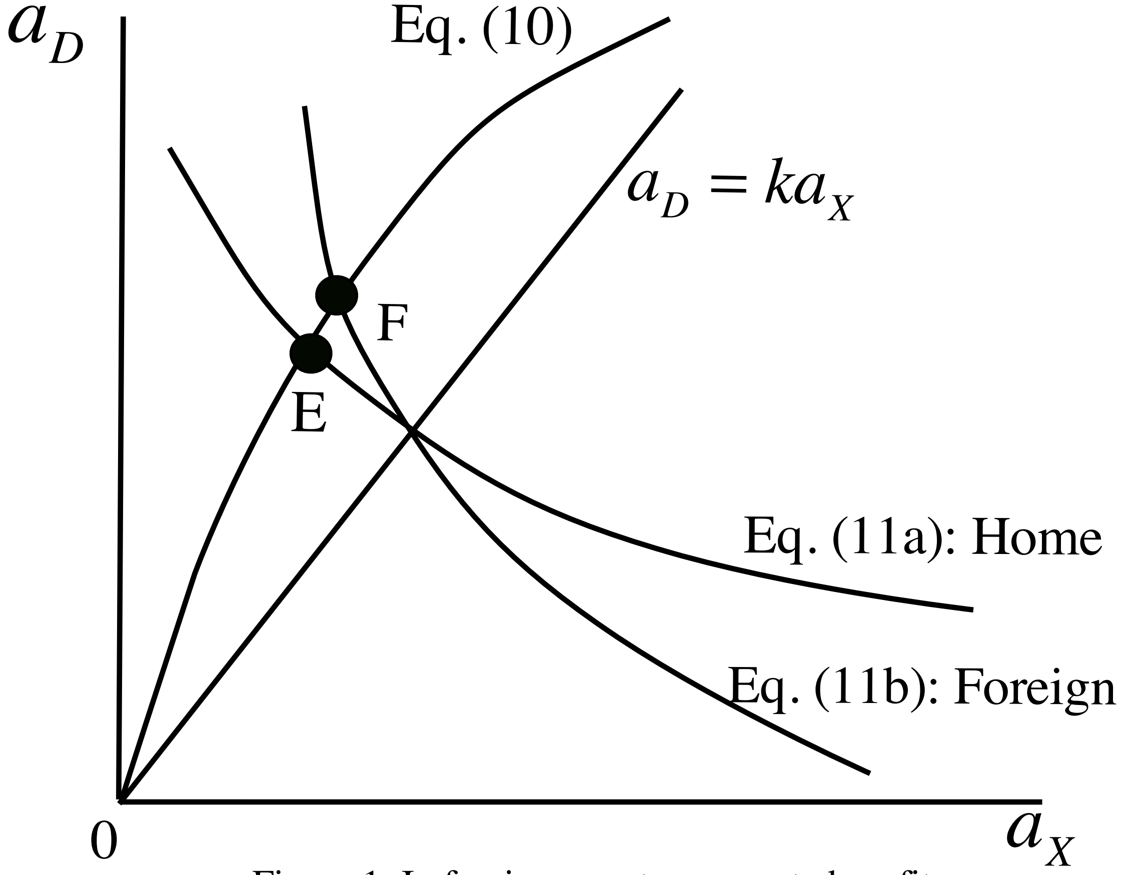

Figure 1: In foreign country, expected profit is less than entry cost (when $k < k^*$)

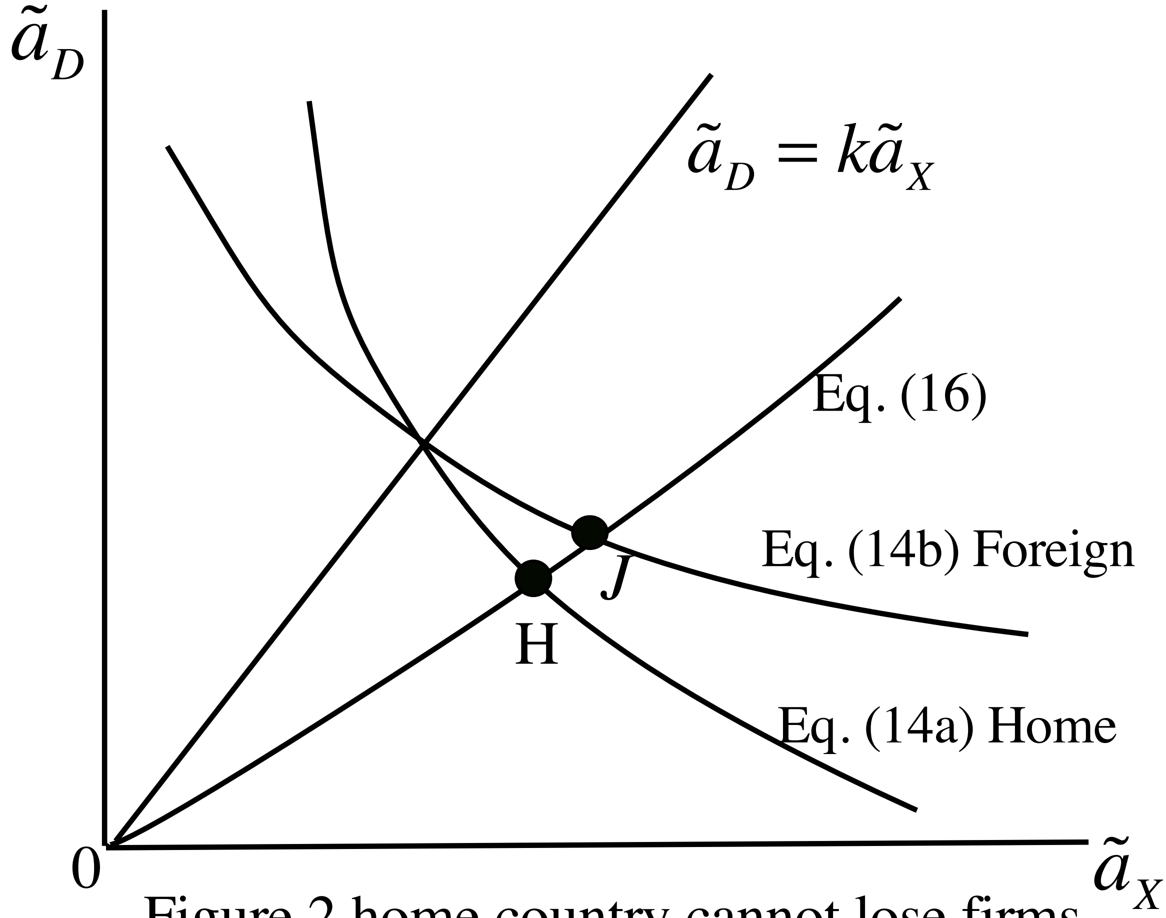

Figure 2 home country cannot lose firms